Approximate Coordinate Transformations for Simulation of Turbulent Flows with Wall Deformation

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In the present paper, approximate coordinate transformations for simulation of turbulent flows with wall deformation, significantly reducing computational cost with little degradation in numerical accuracy, are presented. The Navier-Stokes equations are coordinate-transformed with an approximation of Taylor-series truncation. The performance is evaluated by performing numerical simulations of a channel flow at $Re_\tau = 140$ with active wall motions of $|\eta_m^+| \leq 5$. The approximate transformations provide flow structures as well as turbulence statistics in good agreement with those from a complete transformation [Phys. Fluids 12, 3301 (2000)] and allow 25-30% savings in the CPU time as compared to the complete one.

Key Words : Turblence Control, Drag Reduction, Active Wall Motion

Nomenclature

C, W_k	: Control parameters
i	: Imaginary-number unit
P, p	: Pressure
t, x_i	: Time/space
u_o, u_τ	: Laminar-centerline/friction velocity
δ	: Increment
η_u, η_d, η, η_o	: Displacement parameters
τ, ξ_i	: Transformed time/space
h	: Channel half-width
k_i, k	: Wave numbers
S, S_i	: Source terms
u_i	: Velocity
v_i	: Transformed velocity
δ_{ij}	: Kronecker delta function
φ_i, ϕ_i	: Metric coefficients
ν	: Kinematic viscosity

Superscripts

$\wedge$	: Fourier component
$+$	: Wall coordinates

Subscripts

$Re\left(=\frac{u_o h}{\nu}\right)$, $Re_\tau\left(=\frac{u_\tau h}{\nu}\right)$: Reynolds numbers

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1. Introduction

In recent years many attempts have been made to control turbulence in wall-bounded flows, especially with an objective of skin-friction drag reduction (Gad-el-Hak, 1996). Most of such attempts have focused on suppressing or counteracting the near-wall coherent structures which are responsible for turbulence production. Among numerous turbulence control strategies investigated so far, an active wall motion is one of the most practical control ways for skin-friction drag reduction. Nevertheless, little research on the active wall motion has been performed.

Carlson and Lumley (1996) investigated the efficacy of wall motion by performing direct numerical simulation (DNS) of minimal turbulent channel flow. The channel contains one pair of coherent structures near the wall and one actuator, Gaussian in shape and 12 wall units in height, is installed on the wall. They found that raising the actuator underneath a low-speed streak increases skin-friction drag by allowing the adjacent high-speed region to expand, and vice versa. Mito and Kasagi (1998) performed DNS of turbulent channel flows with a simple oscillatory mode of wall deformation, uniform in the streamwise direction and sinusoidal in the spanwise direction and in time, and then found that the oscillatory wall motions are effective in altering the skin friction and statistical quantities, but hardly reduce drag.

Recently, Endo *et al.* (2000) and Kang and Choi (2000) successfully investigated the possibility of skin-friction drag reduction with wall deformation in wall-bounded turbulent flows by performing DNS of channel flow. The former study proposed a *feedback wall-deformation control* using wall-variable sensors and arrayed deformable actuators, leading to about 10% drag reduction at $Re_\tau = u_\tau h/\nu = 150$ through selective manipulation of the streamwise vortices and streak meandering. Here, u_τ is the wall-shear velocity, h the channel half-width, and ν the kinematic viscosity. On the other hand, the latter proposed active wall motions based on two successful feedback control strategies (Choi *et al.*, 1994; Lee *et al.*, 1998), leading to 13-17% drag reductions at $Re_\tau = 140$. However, most of such research suffer from severely high computational cost because full metric coefficients for addressing deformed walls have to be computed every time step. Hence, the development of a lower-cost and still accurate coordinate transformation is critical to successful simulation of turbulent flows with wall deformation.

The objective of the present paper is to present approximate coordinate transformations for more effectively predicting turbulent wall-deformation flows. The study is motivated by observations that in real applications the amount of maximum wall deformation is restricted to be very small, as exemplified in Endo *et al.* (2000) and Kang and Choi (2000). The restriction suggests that the Navier-Stokes equations should be coordinate-transformed with an approximation of Taylor series truncation. The performance of the proposed transformations is evaluated by comparing their results with those from the complete transformation in the same numerical simulations as in Kang and Choi (2000). This would lead to a significant reduction in the required computational cost with little degradation in numerical accuracy as compared to the complete one, making turbulent flows with wall deformation more accessible to computation.

2. Numerical Method

The turbulent channel flow to simulate is periodic in the streamwise (x_1) and spanwise (x_3) directions and has two parallel walls apart by $2h$ on average in the wall-normal (x_2) direction, as shown in Fig. 1(a). The governing equations for the velocities u_i and pressure p are written as:

$$\frac{\partial u_i}{\partial t} = -\frac{\partial (u_i u_j)}{\partial x_j} - \frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j \partial x_j} - \frac{dP}{dx_1}\delta_{i1} \quad (1)$$

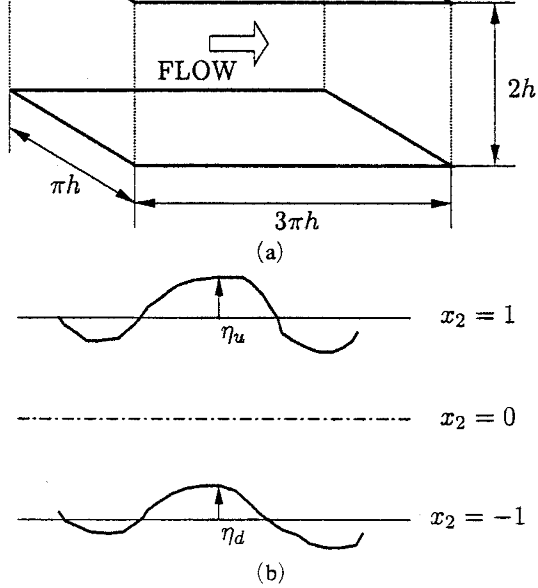


Fig. 1 Channel flow: (a) schematic of the channel and (b) wall deformation

$$\frac{\partial u_i}{\partial x_i}=0 \tag{2}$$

where $-dP/dx_1$ is the mean streamwise pressure gradient (total drag divided by the channel volume) to impose a constant mass flow rate. All variables are nondimensionalized by the channel half-width h and the laminar centerline velocity u_o.

To exactly address the deformed walls, Kang and Choi (2000) introduced a complete coordinate set (τ, ξ_i) as follows:

$$t=\tau,\ x_1=\xi_1,\ x_2=\xi_2(1+\eta)+\eta_o,\ x_3=\xi_3 \tag{3}$$

where $\eta=(\eta_u-\eta_d)/2$ and $\eta_o=(\eta_u+\eta_d)/2$. Here, η_u and η_d, as shown in Fig. 1(b), are respectively the displacements of the upper and lower walls with respect to the uncontrolled state and thus are only functions of variables (t, x_1, x_3). With the coordinate transformation (3), the transformed governing equations can be written as:

$$\frac{\partial u_i}{\partial \tau}=-\frac{\partial(u_iu_j)}{\partial \xi_j}-\frac{\partial p}{\partial \xi_i}+\frac{1}{Re}\frac{\partial^2 u_i}{\partial \xi_j\partial \xi_j}-\frac{dP}{d\xi_1}\delta_{i1}+S_i \tag{4}$$

$$\frac{\partial u_i}{\partial \xi_i}=-S \tag{5}$$

where

$$S_i=-\varphi_t\frac{\partial u_i}{\partial \xi_2}-\phi_j\frac{\partial(u_iu_j)}{\partial \xi_2}-\phi_j\delta_{ij}\frac{\partial p}{\partial \xi_2}+\frac{1}{Re}\left(2\phi_j\frac{\partial^2 u_i}{\partial \xi_j\partial \xi_2}+\phi_j\phi_j\frac{\partial^2 u_i}{\partial \xi_2^2}+\frac{1}{2}\frac{\partial(\phi_j\phi_j)}{\partial \xi_2}\frac{\partial u_i}{\partial \xi_2}\right) \tag{6}$$

$$S=\phi_j\frac{\partial u_j}{\partial \xi_2} \tag{7}$$

$$\phi_j=\varphi_j-\delta_{j2} \tag{8}$$

Here, the metric coefficients, φ's, are computed simply as follows:

$$\begin{cases} \varphi_t=\dfrac{\partial \xi_2}{\partial t}=-\dfrac{1}{1+\eta}\left(\xi_2\dfrac{\partial \eta}{\partial_\tau}+\dfrac{\partial \eta_o}{\partial_\tau}\right) \\ \varphi_j=\dfrac{\partial \xi_2}{\partial x_j}=-\dfrac{1}{1+\eta}\left(\xi_2\dfrac{\partial \eta}{\partial \xi_j}+\dfrac{\partial \eta_o}{\partial \xi_j}\right)\ (j\neq 2) \\ \varphi_2=\dfrac{\partial \xi_2}{\partial x_2}=\dfrac{1}{1+\eta} \end{cases} \tag{9}$$

Kang and Choi (2000) successfully applied the transformed Eqs. (4) and (5) to simulation of a channel flow at $Re_\tau=140$ with active wall motions of $|\eta_m^+|\leq 5(m=u, d)$. They found that, with such small wall deformation, the characteristics of turbulence inclusive of drag reduction is hardly influenced by the wall geometry itself. This suggests that the terms dominating the effect of the wall geometry can be neglected in the derivation of the transformed governing equations without a severe loss of accuracy. Therefore, we assume that the metric coefficients (9) should be approximated, based on the database of Kang and Choi (2000), as

$$\varphi_t\approx 0,\ \varphi_1\approx 0,\ \varphi_2\approx 1-\eta,\ \varphi_3\approx 0. \tag{10}$$

The first two assumptions are enough reasonable because the wall shapes are slow in changing and elongated in the streamwise direction. On the other hand, the last one is marginal because the value is not negligibly small, for example $\varphi_3|\,\mathrm{rms}\approx 0.20$, which has to be evaluated through the simulation result afterwards. The third one is derived using the assumption of small wall-deformation magnitudes in conjunction with an approximation of the Taylor-series truncation. The validity of the Taylor-series truncation at $|\eta_m^+|\leq 5$ is indirectly confirmed by comparing the velocity at $y^+\simeq 5$ with the Taylor-series truncated value about the wall, showing their correlation coefficient of 0.96. With the assumptions (10), the source terms (6) and (7) are replaced with

$$S_i\approx\frac{\partial(\eta u_iu_2)}{\partial \xi_2}+\frac{\partial(\eta p)}{\partial \xi_2}\delta_{i2}-\frac{2\eta}{Re}\frac{\partial^2 u_i}{\partial \xi_2^2} \tag{11}$$

$$S\approx-\frac{\partial(\eta u_2)}{\partial \xi_2} \tag{12}$$

The governing Eqs. (4) and (5) with (11) and (12) are directly solved for simulation of channel flow with active wall deformations.

The approximate governing equations can also be written in another form as follows, by introducing a new variable set (v_i) such that

$$u_1=v_1,\ u_2=v_2(1+\eta),\ u_3=v_3 \tag{13}$$

and then neglecting the higher-order terms of η:

$$\frac{\partial v_i}{\partial \tau}\approx-\frac{\partial(v_iv_j)}{\partial \xi_j}-\frac{\partial p}{\partial \xi_i}+\frac{1}{Re}\frac{\partial^2 v_i}{\partial \xi_j\partial \xi_j}-\frac{dP}{d\xi_1}+\delta_{i1}+S_i' \tag{14}$$

$$\frac{\partial v_i}{\partial \xi_i}\approx 0 \tag{15}$$

where

$$S_i'=-\frac{2\eta}{Re}\frac{\partial^2 v_i}{\partial \xi_2^2}+H\delta_{i2} \tag{16}$$

$$H=-\frac{\partial(\eta v_2)}{\partial\tau}-\frac{\partial(\eta v_2 v_j)}{\partial\xi_j}+\frac{\partial(\eta p)}{\partial\xi_2}+\frac{1}{Re}\frac{\partial^2(\eta v_2)}{\partial\xi_j\partial\xi_j} \quad (17)$$

Until now, we have two sets of approximately-transformed governing equations derived with little difference from the same assumptions (10). Note that the former governing equations (Eqs. (4) and (5) with (11) and (12); u_i-approximate transformation) have fewer terms, whereas the latter ones (Eqs. (14) and (15) with (16) and (17); v_i-approximate transformation) do not have a source term in the continuity equation.

The mean pressure gradients are obtained by computing $-dP/d\xi_1=-\int b^* d\xi^3 / \int (1/\varphi_2)\, d\xi^3$, where

$$\begin{cases} b^*=u_1\dfrac{\partial\eta}{\partial\tau}+\dfrac{1}{\varphi_2}\left(-\dfrac{\partial(u_1u_j)}{\partial\xi_j}-\dfrac{\partial p}{\partial\xi_1}+\dfrac{1}{Re}\dfrac{\partial^2 u_1}{\partial\xi_j\partial\xi_j}+S_1\right) & (u_i\text{-approx.}) \\ b^*=v_1\dfrac{\partial\eta}{\partial\tau}+\dfrac{1}{\varphi_2}\left(-\dfrac{\partial(v_1v_j)}{\partial\xi_j}-\dfrac{\partial p}{\partial\xi_1}+\dfrac{1}{Re}\dfrac{\partial^2 v_1}{\partial\xi_j\partial\xi_j}+S'_1\right) & (v_i\text{-approx.}) \end{cases} \quad (18)$$

and the integral is taken over the whole computational domain ($d\xi^3=d\xi_1 d\xi_2 d\xi_3$).

The drag reduction is attempted by locally deforming the wall based on two successful feedback control strategies. That is, the wall is locally deformed such that the velocity induced by the wall motion is equal to (i) the wall-normal velocity at $x_2^+\simeq 10$ with an opposite sign (u_2-control; Choi *et al.*, 1994) or (ii) the suboptimal wall actuation proportional to the spanwise derivative of the spanwise-velocity gradient at the wall (du_3/dx_2-control; Lee *et al.*, 1998). The boundary conditions, for example η_d, become

$$\begin{cases} u_1=u_3=0,\ u_2=\dfrac{\partial\eta_d}{\partial\tau} \text{ at } \xi_2=-1 & (u_i-\text{approx.}) \\ v_1=v_3=0,\ v_2=\varphi_2\dfrac{\partial\eta_d}{\partial\tau} \text{ at } \xi_2=-1 & (v_i-\text{approx.}) \end{cases} \quad (19)$$

where

$$\begin{cases} \dfrac{\partial\eta_d}{\partial t}=-\dfrac{1}{\varphi_2}v_2\,|_{y^+\simeq 10} & (u_2-\text{control}) \\ \dfrac{\partial\hat{\eta}_d}{\partial t}=C\dfrac{ik_3}{k}\varphi_2\dfrac{\partial v_3}{\partial\xi_2}\,|_{\xi_2=-1} & \left(\dfrac{du_3}{dx_2}-\text{control}\right) \end{cases} \quad (20)$$

Here, x_2^+ is the wall-normal distance in wall unit, $k=\sqrt{k_1^2+k_3^2}$ where k_1 and k_3 are respectively the streamwise and spanwise wavenumbers, and C a constant. The superscript hat($\wedge$) denotes the Fourier component.

The numerical method used in the simulations is based on a semi-implicit, fractional step method: the diffusion and nonlinear terms are advanced in time with the Crank-Nicolson method and a third-order Runge-Kutta method, respectively. All spatial derivatives are discretized with a second-order central-difference scheme.

3. Results

The performance of the proposed approximate coordinate transformations is evaluated by performing DNS of the same channel flow as in Kang and Choi (2000) and comparing the results with those obtained from the complete coordinate transformation. Starting from an uncontrolled fully-developed turbulent channel flow at $Re_\tau=140$ ($Re=u_o h/\nu=3000$), numerical simulations are performed with a resolution of $64\times97\times96$ ($x_1\times x_2\times x_3$) on a computational box of $3\pi h\times 2h\times\pi h$ and $\Delta t=0.05u_o/h$, corresponding to $\Delta x_1^+\approx 20$ and $\Delta x_3^+\approx 4.5$ and $\Delta t^+\approx 0.32$, respectively. Notice that all variables in wall units used hereinafter are based on the no-control friction velocity u_τ. In all the simulations, the amount of the maximum wall deformation is restricted to be $|\eta_m^+|\leq 5$.

Figure 2 shows time traces of the mean streamwise pressure gradient for two control strategies (u_2- and du_3/dx_2-controls) and two approximate coordinate transformations (v_i- and u_i-approximations) compared with the complete results (Kang and Choi, 2000). Results show that the approximate transformations predict an overall 14-16% drag reduction compared to 13-17% from the complete predictions, showing the good performance of the proposed transformations. Such agreement can also be found in the predictions of other turbulence quantities (not shown here). Consequently, it can be shown that the assumption (10), especially $\varphi_3\approx 0$, hardly corrupts the simulation results and thus is fairly acceptable.

Subsequently, the comparison of performance between the two approximate transformations has to be made to validate their derivation. Figure 2 shows clearly that there is little difference on average between the two results, despite a small difference at every moment due to the intrinsic turbulence instability. It can be explained by the fact that v_i-approximate transformation differs from u_i-approximate one, by several terms of order higher than η^2 which are negligible owing to the small amount of wall deformation. This means that the two approximate transformations can be applied interchangeably with little performance difference in simulation of turbulent flows with such active wall motions. In the present paper, therefore, only the v_i-approximate results are hereinafter presented.

The approximate transformations have to be also assessed in terms of CPU time. With the active wall motion of u_2-control, v_i-approximate transformation requires about 2.7 sec per time step on CRAY YMP C90 while the complete one requires 3.7 sec, showing that the former allows 25-30% savings in CPU time as compared to the latter. Therefore, it is concluded that the proposed approximate transformations are promising tools for simulation of wall-motion flows.

Figure 3 shows the contours of the instantaneous streamwise vorticity in a cross-flow plane during control, together with the corresponding deformed walls. The figure proves that the active wall motions significantly weaken the strength of the streamwise vorticity near the wall responsible

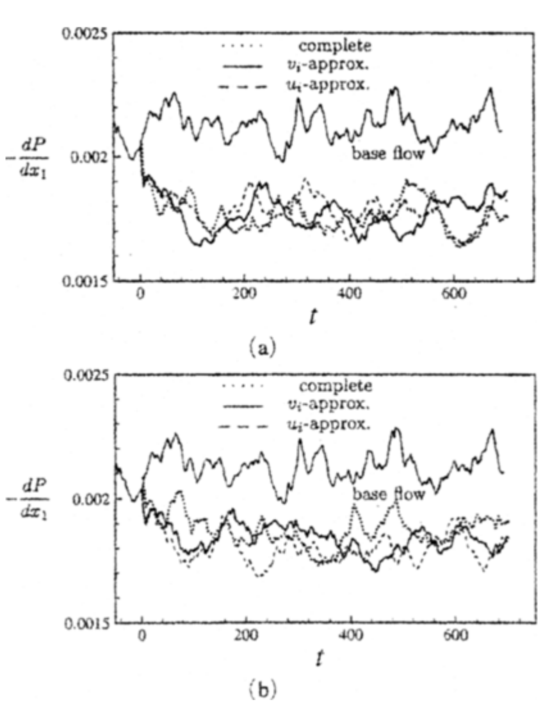


Fig. 2 Time traces of the mean streamwise pressure gradient: (a) u_2-control; (b) du_3/dx_2-control

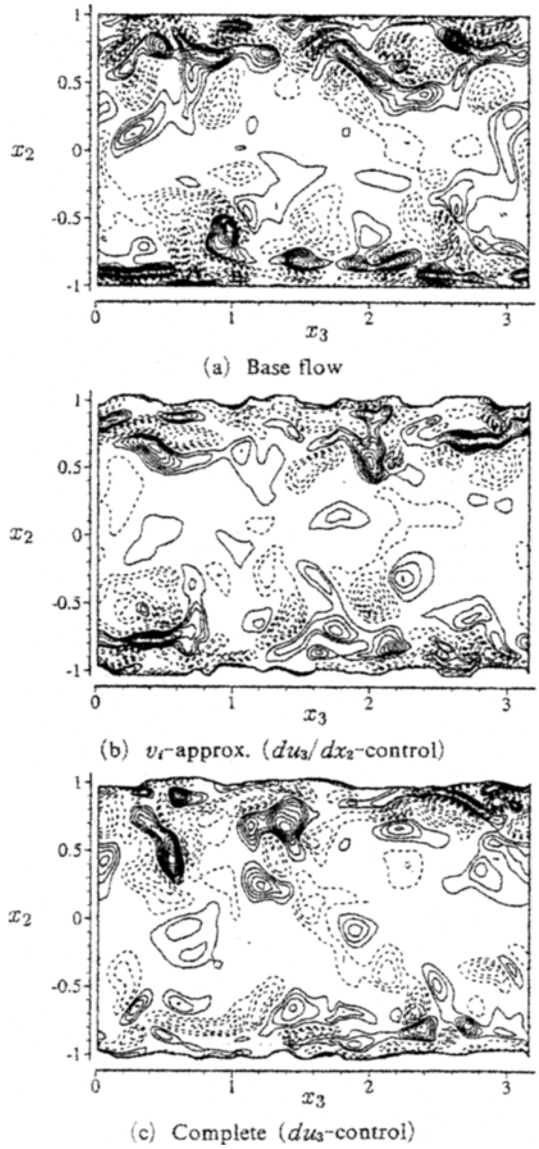
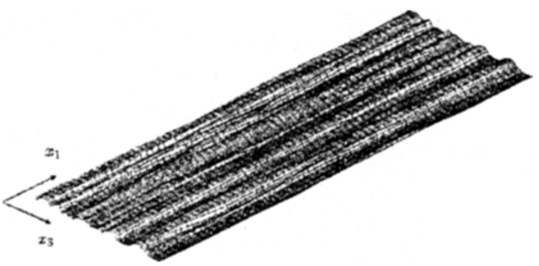


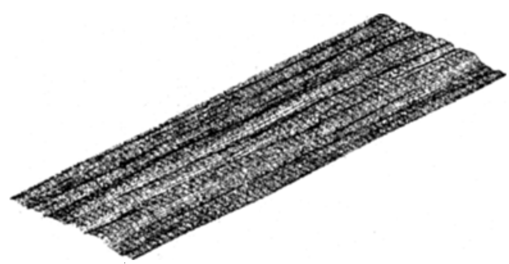
Fig. 3 Contours of the instantaneous streamwise vorticity in a cross-flow plane for du_3/dx_2-control. Contour levels of $\omega_1 h/u_o$ are from -3 to 3 by increments of 0.2. Negative values are shown dashed

for the production of turbulence, leading to drag reduction. The result agrees well to the complete one, indicating that the proposed approximate transformations are promising in predicting wall shapes and turbulence structures.

Figures 4 and 5 show respectively the instantaneous wall shapes and contours of the wall-normal velocity at the wall for both active wall motions. The flattened tops and bottoms of the ribs and grooves in the figure illustrate the saturation phenomena of wall motion which occur because of the limited wall deformation. The wall shapes are elongated in the streamwise direction and resemble riblets in appearance, even though the contours of the instantaneous wall-normal velocity at the wall are intermittent and oval-shaped as shown in Fig. 5. The patterns in the wall shape and wall-normal velocity are in good agreement with the complete results. Despite the good performance, however, a little discrepancy can be found according to the approximation. In the present simulations, the average rms values of wall deformation is predicted to remain around 3.5 in wall units, which is slightly higher than the complete result of 3.2 in wall units.



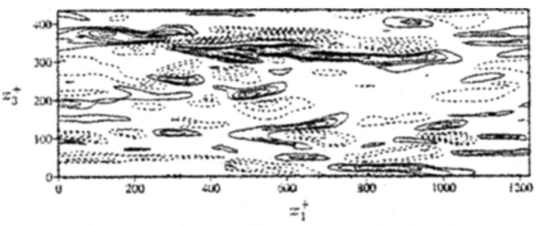
(a) v_i-approx. (du_3/dx_2-control)



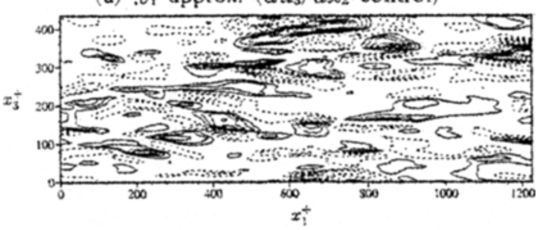
(b) Complete (du_3/dx_2-control)

Fig. 4 Instantaneous shapes of wall deformation during control for du_3/dx_2-control

In order to quantitatively analyze the riblet-like structures at the wall, spanwise two-point correlations of wall deformation are shown in Fig. 6 and compared with the complete results. The average spanwise spacing of the wall peak-to-peak is predicted about 85-95 in wall units for u_2-control and 100-110 for du_3/dx_2-control, with the latter slightly larger than the former. The results are comparable to the complete ones, proving once again the good performance of the proposed approximate transformations in predicting the active wall motions.



(a) v_i-approx. (du_3/dx_2-control)



(b) Complete (du_3/dx_2-control)

Fig. 5 Contours of the wall-normal velocity at the wall for du_3/dx_2-control. Contour levels of u_2/u_τ are from −0.96 to 0.96 by increments of 0.08. Negative values are shown dashed

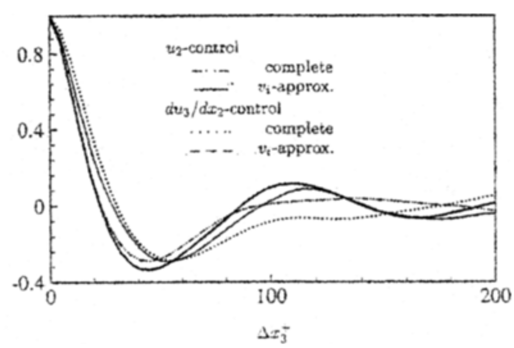


Fig. 6 Two-point correlation coefficients of the wall deformation in the spanwise direction

4. Summary

In general, numerical simulations of wall-bounded turbulent flows with wall deformation requires grids to be set up at every time step. This makes the computations prohibitively costly because full metric coefficients have to be computed at each time. This paper, therefore, suggests new approximate coordinate transformations for effectively addressing active wall motions with little loss in numerical accuracy. The Navier-Stokes equations are coordinate-transformed with an approximation of Taylor-series truncation. The performance was evaluated by performing numerical simulations of the same channel flow with active wall motions at $Re_\tau=$ 140 as in Kang and Choi (2000). The drag reduction was attempted by locally deforming the wall based on two successful feedback control strategies suggested by Choi *et al.* (1994) and Lee et al. (1998) and the amount of the maximum wall deformation was restricted to be 5 in wall units.

The results were compared with those obtained by the complete coordinate transformation for the same flow (Kang and Choi, 2000). The comparison showed that the approximate transformations predicted drag reductions, turbulent statistics, and flow structures in good agreement with the complete results and allowed 25-30% savings in the CPU time as compared to the complete one. This would make the proposed approximate transformations promising in the simulations of turbulent flows with wall deformation.

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